QUESTION BOOKLET – 2017
Subject : Paper I : Mathematics

Question Booklet Version

11
(Write this number on your Answer Sheet)

Roll No.

Question Booklet Sr. No.

1008142
(Write this number on your Answer Sheet)

Answer Sheet No.

Duration: 1 Hour 30 Minutes

Total Marks : 100

This is to certify that, the entries of Roll Number and Answer Sheet Number have been correctly written and verified.

Candidate’s Signature

Invigilator’s Signature

Instructions to Candidates

1. This question booklet contains 50 Objective Type Questions (Single Best Response Type) in the subject of Mathematics.
2. The question paper and OMR (Optical Mark Reader) Answer Sheet are issued to examinees separately at the beginning of the examination session.
3. Choice and sequence for attempting questions will be as per the convenience of the candidate.
4. Candidate should carefully read the instructions printed on the Question Booklet and Answer Sheet and make the correct entries on the Answer Sheet. As Answer Sheets are designed to suit the OPTICAL MARK READER (OMR) SYSTEM, special care should be taken to mark appropriate entries/answers correctly. Special care should be taken to fill QUESTION BOOKLET VERSION, SERIAL NO. and Roll No. accurately. The correctness of entries has to be cross-checked by the invigilators. The candidate must sign on the Answer Sheet and Question Booklet.
5. Read each question carefully.
6. Determine the correct answer from out of the four available options given for each question.
7. Fill the appropriate circle completely like this o, for answering the particular question, with Black ink ball point pen only, in the OMR Answer Sheet.
8. Each answer with correct response shall be awarded two (2) marks. There is no Negative Marking. If the examinee has marked two or more answers or has done scratching and overwriting in the Answer Sheet in response to any question, or has marked the circles inappropriately e.g. half circle, dot, tick mark, cross etc. mark/s shall NOT be awarded for such answer/s, as these may not be read by the scanner. Answer sheet of each candidate will be evaluated by computerized scanning method only (Optical Mark Reader) and there will not be any manual checking during evaluation or verification.
9. Use of whitener or any other material to erase/hide the circle once filled is not permitted. Avoid overwriting and/or striking of answers once marked.
10. Rough work should be done only on the blank space provided in the Question Booklet. Rough work should not be done on the Answer Sheet.
11. The required mathematical tables (Log etc.) are provided within the question booklet.
12. Immediately after the prescribed examination time is over, the Answer Sheet is to be returned to the Invigilator. Confirm that both the Candidate and Invigilator have signed on question booklet and answer sheet.
13. No candidate is allowed to leave the examination hall till the examination session is over.
1. The statement pattern \((\neg p \land q)\) is logically equivalent to
   A) \((p \lor q) \lor \neg p\)
   C) \((p \land q) \rightarrow p\)
   B) \((p \lor q) \land \neg p\)
   D) \((p \lor q) \rightarrow p\)

2. If \(g(x)\) is the inverse function of \(f(x)\) and \(f'(x) = \frac{1}{1+x^4}\) then \(g'(x)\) is
   A) \(1 + [g(x)]^4\)
   B) \(1 - [g(x)]^4\)
   C) \(1 + [f(x)]^4\)
   D) \(\frac{1}{1+[g(x)]^4}\)

3. The inverse of the matrix \[
\begin{bmatrix}
1 & 0 & 0 \\
3 & 3 & 0 \\
5 & 2 & -1
\end{bmatrix}
\]
is
   A) \[
\frac{1}{3}
\begin{bmatrix}
-3 & 0 & 0 \\
3 & 1 & 0 \\
9 & 2 & -3
\end{bmatrix}
\]
   B) \[
\frac{1}{3}
\begin{bmatrix}
-3 & 0 & 0 \\
3 & -1 & 0 \\
-9 & -2 & 3
\end{bmatrix}
\]
   C) \[
\frac{1}{3}
\begin{bmatrix}
3 & 0 & 0 \\
3 & -1 & 0 \\
-9 & -2 & 3
\end{bmatrix}
\]
   D) \[
\frac{1}{3}
\begin{bmatrix}
-3 & 0 & 0 \\
3 & 1 & 0 \\
-9 & -2 & 3
\end{bmatrix}
\]
4. If \( \int \frac{1}{\sqrt{9-16x^2}} \, dx = \alpha \sin^{-1}(\beta x) + c \) then \( \alpha = \frac{1}{\beta} \)  

A) 1 \hspace{1cm} B) \frac{7}{12} \hspace{1cm} C) \frac{19}{12} \hspace{1cm} D) \frac{9}{12} 

5. O(0, 0), A(1, 2), B(3, 4) are the vertices of \( \triangle OAB \). The joint equation of the altitude and median drawn from O is  

A) \( x^2 + 7xy - y^2 = 0 \) \hspace{1cm} B) \( x^2 + 7xy + y^2 = 0 \) \hspace{1cm} C) \( 3x^2 - xy - 2y^2 = 0 \) \hspace{1cm} D) \( 3x^2 + xy - 2y^2 = 0 \)  

6. If \( \int \frac{1}{(x^2 + 4)(x^2 + 9)} \, dx = A \tan^{-1} \frac{x}{2} + B \tan^{-1} \frac{x}{3} + C \) then \( A - B = \)  

A) \( \frac{1}{6} \) \hspace{1cm} B) \( \frac{1}{30} \) \hspace{1cm} C) \( -\frac{1}{30} \) \hspace{1cm} D) \( -\frac{1}{6} \)  

7. If \( \alpha \) and \( \beta \) are roots of the equation \( x^2 + 5|x| - 6 = 0 \) then the value of \( |\tan^{-1} \alpha - \tan^{-1} \beta| \) is  

A) \( \frac{\pi}{2} \) \hspace{1cm} B) 0 \hspace{1cm} C) \( \pi \) \hspace{1cm} D) \( \frac{\pi}{4} \)  

8. If \( x = a(t - \frac{1}{t}) \), \( y = a(t + \frac{1}{t}) \) where \( t \) be the parameter then \( \frac{dy}{dx} = ? \)  

A) \( \frac{y}{x} \) \hspace{1cm} B) \( \frac{-x}{y} \) \hspace{1cm} C) \( \frac{x}{y} \) \hspace{1cm} D) \( \frac{y}{x} \)
9. The point on the curve \( y = \sqrt{x - 1} \) where the tangent is perpendicular to the line \( 2x + y - 5 = 0 \) is
A) (2, -1)  B) (10, 3)  C) (2, 1)  D) (5, -2)

10. If \( \int \frac{x - 5}{\sqrt{x} - 7} \, dx = A \sqrt{x^2 - 12x + 35} + \log|x - 6 + \sqrt{x^2 - 12x + 35}| + C \) then A =
A) -1  B) 1/2  C) -1/2  D) 1

11. The number of principal solutions of \( \tan 2\theta = 1 \) is
A) One  B) Two  C) Three  D) Four

12. The objective function \( z = 4x_1 + 5x_2 \), subject to \( 2x_1 + x_2 \geq 7 \), \( 2x_1 + 3x_2 \leq 15 \), \( x_2 \leq 3 \), \( x_1, x_2 \geq 0 \) has minimum value at the point
A) On x-axis  B) On y-axis  C) At the origin  D) On the line parallel to x-axis

13. If \( z_1 \) and \( z_2 \) are z co-ordinates of the points of trisection of the segment joining the points A(2, 1, 4), B(-1, 3, 6) then \( z_1 + z_2 = \)
A) 1  B) 4  C) 5  D) 10

14. The maximum value of \( f(x) = \frac{\log x}{x} \) (\( x > 0, x \neq 1 \)) is
A) e  B) \( \frac{1}{e} \)  C) \( e^2 \)  D) \( \frac{1}{e^2} \)
9. The point on the curve \( y = \sqrt{x - 1} \) where the tangent is perpendicular to the line \( 2x + y - 5 = 0 \) is
   \[ \text{A) } (2, -1) \quad \text{B) } (10, 3) \quad \text{C) } (2, 1) \quad \text{D) } (5, -2) \]

10. If \( \int \frac{x - 5}{\sqrt{x - 7}} \, dx = A \sqrt{x^2 - 12x + 35} + \log |x - 6 + \sqrt{x^2 - 12x + 35}| + C \) then \( A = \)
   \[ \text{A) } -1 \quad \text{B) } \frac{1}{2} \quad \text{C) } -\frac{1}{2} \quad \text{D) } 1 \]

11. The number of principal solutions of \( \tan 2\theta = 1 \) is
   \[ \text{A) One} \quad \text{B) Two} \quad \text{C) Three} \quad \text{D) Four} \]

12. The objective function \( z = 4x_1 + 5x_2 \), subject to \( 2x_1 + x_2 \geq 7 \), \( 2x_1 + 3x_2 \leq 15 \), \( x_2 \leq 3 \), \( x_1, x_2 \geq 0 \) has minimum value at the point
   \[ \text{A) On x-axis} \quad \text{B) On y-axis} \quad \text{C) At the origin} \quad \text{D) On the line parallel to x-axis} \]

13. If \( z_1 \) and \( z_2 \) are the co-ordinates of the points of trisection of the segment joining the points \( A(2, 1, 4) \), \( B(-1, 3, 6) \) then \( z_1 + z_2 = \)
   \[ \text{A) } 1 \quad \text{B) } 4 \quad \text{C) } 5 \quad \text{D) } 10 \]

14. The maximum value of \( f(x) = \frac{\log x}{x} \) \( (x > 0, x \neq 1) \) is
   \[ \text{A) } e \quad \text{B) } \frac{1}{e} \quad \text{C) } e^2 \quad \text{D) } \frac{1}{e^2} \]

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**SPACE FOR ROUGH WORK**
15. $\int_0^1 \tan^{-1} x \, dx =$
   
   A) $\frac{\pi}{4} + \frac{1}{2}$  
   B) $\frac{\pi}{4} - \frac{1}{2}$  
   C) $\frac{1}{2} - \frac{\pi}{4}$  
   D) $-\frac{\pi}{4} - \frac{1}{2}$

16. If $c$ denotes the contradiction then dual of the compound statement $p \land (q \lor c)$ is
   
   A) $p \lor (q \land t)$  
   B) $p \land (q \lor t)$  
   C) $p \lor (q \land t)$  
   D) $p \lor (q \land t)$

17. The differential equation of all parabolas whose axis is $y$-axis is
   
   A) $x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$  
   B) $x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$  
   C) $\frac{d^2y}{dx^2} - y = 0$  
   D) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

18. $\int_0^3 [x] \, dx =$
   
   A) 3  
   B) 0  
   C) 2  
   D) 1

19. The objective function of LPP defined over the convex set attains its optimum value at
   
   A) At least two of the corner points  
   B) All the corner points  
   C) At least one of the corner points  
   D) None of the corner points

   $\begin{bmatrix} \alpha & 14 & -1 \\ 2 & 3 & 1 \\ 6 & 2 & 3 \end{bmatrix}$

20. If the inverse of the matrix $\begin{bmatrix} \alpha & 14 & -1 \\ 2 & 3 & 1 \\ 6 & 2 & 3 \end{bmatrix}$ does not exist then the value of $\alpha$ is
   
   A) 1  
   B) $-1$  
   C) 0  
   D) $-2$

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SPACE FOR ROUGH WORK
21. If the function \( f(x) = \left[ \tan \left( \frac{\pi}{4} + x \right) \right]^{\frac{x}{2}} \) for \( x \neq 0 \)
\[ = K \quad \text{for} \quad x = 0 \]
is continuous at \( x = 0 \) then \( K = ? \)
A) \( e \) \quad B) \( e^{-1} \) \quad C) \( e^{2} \) \quad D) \( e^{-2} \)

22. For a invertible matrix \( A \) if \( A(\text{adj} \, A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \) then \( |A| = \)
A) 100 \quad B) -100 \quad C) 10 \quad D) -10

23. The solution of the differential equation \[ \frac{dy}{dx} = \tan \left( \frac{y}{x} \right) + \frac{y}{x} \]
is
A) \( \cos \left( \frac{y}{x} \right) = cx \) \quad B) \( \sin \left( \frac{y}{x} \right) = cx \)
C) \( \cos \left( \frac{y}{x} \right) = cy \) \quad D) \( \sin \left( \frac{y}{x} \right) = cy \)

24. In \( \triangle ABC \) if \( \sin^2 A + \sin^2 B = \sin^2 C \) and \( l(AB) = 10 \) then the maximum value of the area of \( \triangle ABC \) is
A) 50 \quad B) 10\sqrt{2} \quad C) 25 \quad D) 25\sqrt{2}
25. If \( x = f(t) \) and \( y = g(t) \) are differentiable functions of \( t \) then \( \frac{d^2y}{dx^2} \) is

\[
\begin{align*}
A) & \quad \frac{f'(t)g''(t) - g'(t)f''(t)}{[f'(t)]^3} \\
B) & \quad \frac{f'(t)g''(t) - g'(t)f''(t)}{[f'(t)]^2} \\
C) & \quad \frac{g'(t)f''(t) - f'(t)g''(t)}{[f'(t)]^3} \\
D) & \quad \frac{g'(t)f''(t) + f'(t)g''(t)}{[f'(t)]^3}
\end{align*}
\]

26. A r.v. \( X \sim B(n, \ p) \). If values of mean and variance of \( X \) are 18 and 12 respectively then total number of possible values of \( X \) are

\[
\begin{align*}
\text{A)} & \quad 54 \\
\text{B)} & \quad 55 \\
\text{C)} & \quad 12 \\
\text{D)} & \quad 18
\end{align*}
\]

27. The area of the region bounded by the lines \( y = 2x + 1, \ y = 3x + 1 \) and \( x = 4 \) is

\[
\begin{align*}
\text{A)} & \quad 16 \text{ sq. unit} \\
\text{B)} & \quad \frac{121}{3} \text{ sq. unit} \\
\text{C)} & \quad \frac{121}{6} \text{ sq. unit} \\
\text{D)} & \quad 8 \text{ sq. unit}
\end{align*}
\]

28. A box contains 6 pens, 2 of which are defective. Two pens are taken randomly from the box. If r.v. \( X \) : Number of defective pens obtained, then standard deviation of \( X = \)

\[
\begin{align*}
\text{A)} & \quad \pm \frac{4}{3\sqrt{5}} \\
\text{B)} & \quad \frac{8}{3} \\
\text{C)} & \quad \frac{16}{45} \\
\text{D)} & \quad \frac{4}{3\sqrt{5}}
\end{align*}
\]
29. If the volume of a spherical ball is increasing at the rate of $4\pi$ cc/sec then the rate of change of its surface area when the volume is $288\pi$ cc is

- A) $\frac{4}{3}\pi$ cm$^2$/sec
- B) $\frac{2}{3}\pi$ cm$^2$/sec
- C) $4\pi$ cm$^2$/sec
- D) $2\pi$ cm$^2$/sec

30. If $f(x) = \log(\sec^2 x) \cot^2 x$, for $x \neq 0$

$$= K$$

for $x = 0$

is continuous at $x = 0$ then $K$ is

- A) $e^{-1}$
- B) 1
- C) $e$
- D) 0

31. If the origin and the points P(2, 3, 4), Q(1, 2, 3) and R(x, y, z) are co-planar then

- A) $x - 2y - z = 0$
- B) $x + 2y + z = 0$
- C) $x - 2y + z = 0$
- D) $2x - 2y + z = 0$

32. If lines represented by equation $px^2 - qy^2 = 0$ are distinct then

- A) $pq > 0$
- B) $pq < 0$
- C) $pq = 0$
- D) $p + q = 0$

33. Let PQRS be a quadrilateral. If M and N are the midpoints of the sides PQ and RS respectively then $\overline{PS} + \overline{QR} =$

- A) $3\overline{MN}$
- B) $4\overline{MN}$
- C) $2\overline{MN}$
- D) $2\overline{NM}$

34. If slopes of lines represented by $Kx^2 + 5xy + y^2 = 0$ differ by 1 then $K =$

- A) 2
- B) 3
- C) 6
- D) 8
35. If vector \( \vec{r} \) with d.c.s. 1, m, n is equally inclined to the co-ordinate axes, then the total number of such vectors is
   A) 4  B) 6  C) 8  D) 2

36. The particular solution of the differential equation \( xdy + 2ydx = 0 \), when \( x = 2, y = 1 \) is
   A) \( xy = 4 \)  B) \( x^2y = 4 \)  C) \( xy^2 = 4 \)  D) \( x^2y^2 = 4 \)

37. \( \Delta ABC \) has vertices at \( A = (2, 3, 5), B = (-1, 3, 2) \) and \( C = (\lambda, 5, \mu) \). If the median through \( A \) is equally inclined to the axes, then the values of \( \lambda \) and \( \mu \) respectively are
   A) 10, 7  B) 9, 10  C) 7, 9  D) 7, 10

38. For the following distribution function \( F(x) \) of a r.v. \( X \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F(x) )</td>
<td>0.2</td>
<td>0.37</td>
<td>0.48</td>
<td>0.62</td>
<td>0.85</td>
<td>1</td>
</tr>
</tbody>
</table>

\( P(3 < x \leq 5) = \)
   A) 0.48  B) 0.37  C) 0.27  D) 1.47

39. The lines \( \frac{x-1}{2} = \frac{y+1}{2} = \frac{z-1}{4} \) and \( \frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} \) intersect each other at point
   A) \((-2, -4, 5)\)  B) \((-2, -4, -5)\)  C) \((2, 4, -5)\)  D) \((2, -4, -5)\)

40. \( \int \frac{\sec^5 x}{\cosec x} \ dx = \)
   A) \( \frac{\sec^6 x}{8} + c \)  B) \( \frac{\sec^7 x}{7} + c \)  C) \( \frac{\sec^6 x}{6} + c \)  D) \( \frac{\sec^9 x}{9} + c \)
41. The equation of line equally inclined to co-ordinate axes and passing through
(-3, 2, -5) is
A) \( \frac{x + 3}{1} = \frac{y - 2}{1} = \frac{z + 5}{1} \)  
B) \( \frac{x + 3}{-1} = \frac{y - 2}{1} = \frac{5 + z}{-1} \)
C) \( \frac{x + 3}{-1} = \frac{y - 2}{1} = \frac{z + 5}{1} \)  
D) \( \frac{x + 3}{-1} = \frac{2 - y}{1} = \frac{z + 5}{-1} \)

42. If \( \int_{0}^{\pi/2} \log \cos x \, dx = \frac{\pi}{2} \log \left( \frac{1}{2} \right) \) then \( \int_{0}^{\pi/2} \log \sec x \, dx = \)
A) \( \frac{\pi}{2} \log \left( \frac{1}{2} \right) \)  
B) \( 1 - \frac{\pi}{2} \log \left( \frac{1}{2} \right) \)  
C) \( 1 + \frac{\pi}{2} \log \left( \frac{1}{2} \right) \)  
D) \( \frac{\pi}{2} \log 2 \)

43. A boy tosses fair coin 3 times. If he gets ₹2X for X heads then his expected gain equals to ₹ . . .
A) 1  
B) \( \frac{3}{2} \)  
C) 3  
D) 4

44. Which of the following statement pattern is a tautology ?
A) \( p \lor (q \rightarrow p) \)  
B) \( \sim q \rightarrow \sim p \)
C) \( (q \rightarrow p) \lor (\sim p \leftrightarrow q) \)  
D) \( p \land \sim p \)

45. If the angle between the planes \( \vec{r} \cdot (\vec{m} \vec{i} - \vec{j} + 2\vec{k}) + 3 = 0 \) and \( \vec{r} \cdot (2\vec{i} - \vec{m} \vec{j} - \vec{k}) - 5 = 0 \) is
\( \frac{\pi}{3} \) then \( m = \)
A) 2  
B) \( \pm 3 \)  
C) 3  
D) -2
46. If \( f(x) = x \) for \( x \leq 0 \)
    \( = 0 \) for \( x > 0 \) then \( f(x) \) at \( x = 0 \) is
   A) Continuous but not differentiable
   B) Not continuous but differentiable
   C) Continuous and differentiable
   D) Not continuous and not differentiable

47. The equation of the plane through \( (-1, 1, 2) \), whose normal makes equal acute angles with co-ordinate axes is
   A) \( \mathbf{r} \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = 2 \)
   B) \( \mathbf{r} \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = 6 \)
   C) \( \mathbf{r} \cdot (3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}) = 2 \)
   D) \( \mathbf{r} \cdot (\mathbf{i} - \mathbf{j} + \mathbf{k}) = 3 \)

48. Probability that a person will develop immunity after vaccination is 0.8. If 8 people are given the vaccine then probability that all develop immunity is =
   A) \( (0.2)^8 \)
   B) \( (0.8)^8 \)
   C) \( 1 \)
   D) \( \binom{8}{6} (0.2)^6 (0.8)^2 \)

49. If the distance of points \( 2\mathbf{i} + 3\mathbf{j} + \lambda\mathbf{k} \) from the plane \( \mathbf{r} \cdot (3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}) = 13 \) is 5 units then \( \lambda = \)
   A) \( 6, -\frac{17}{3} \)
   B) \( 6, \frac{17}{3} \)
   C) \( -6, -\frac{17}{3} \)
   D) \( -6, \frac{17}{3} \)

50. The value of \( \cos^{-1}\left(\cot\left(\frac{\pi}{2}\right)\right) + \cos^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) \) is
   A) \( 0, \frac{2\pi}{3} \)
   B) \( \frac{\pi}{3} \)
   C) \( \frac{\pi}{2} \)
   D) \( \pi \)

SPACE FOR ROUGH WORK