Arithmetic Question & Answers

Question: The smaller of 99^{100} + 100^{100} and 101^{100}, is

(a) 99^{100} + 100^{100}

(b) Both are equal

(c) 101^{100}

(d) None of these

Solution: (a)

$$99^{100} + 100^{100} - 101^{100} = (100-1)^{100} - (100+1)^{100} + 100^{100}$$
$$= -\left[(100+1)^{100} - (100-1)^{100}\right] + 100^{100}$$
$$= -2\left[{}^{100}C_1 100 + {}^{100}C_3 (100)^3 + \ldots + {}^{100}C_{99} (100)^{99}\right] + 100^{100} < 0$$
$$= 99^{100} + 100^{100} - 101^{100} < 100$$
$$= 99^{100} + 100^{100} < 101^{100}$$

Question:

If three successive terms of a G.P with common ratio r ($r > 1$) form the sides of a triangle ABC and $[r]$ denotes greatest integer function, then $[r] + [-r]$ is

(a) 0

(b) 1

(c) −1

(d) None of these

Solution: (c)

Let us suppose the sides of the triangle be a, ar and ar^2 (being the largest side as $r > 1$).

Now, $a + ar > ar^2$ (Sum of two sides is greater than the third side)

$$\Rightarrow ar^2 - ar - a < 0$$
$$\Rightarrow r^2 - r - 1 < 0$$
$$\Rightarrow \frac{1-\sqrt{5}}{2} < r < \frac{1+\sqrt{5}}{2}$$
$$\Rightarrow 1 < r < \frac{1+\sqrt{5}}{2} \quad [\because r > 1]$$
$$\Rightarrow [r] = 1$$

Also, $-\frac{1+\sqrt{5}}{2} < -r < -1$

$$\Rightarrow [-r] = -2$$
$$\therefore [r] + [-r] = 1 - 2 = -1$$

Question:

Eighteen guests have to be seated, half on each side a long table. Four particular guest desires to sit on one particular side and three others on the other side. The number of ways in which the seating arrangement can be made, is

(a) $9! \times 9!$

(b) ${}^{11}C_5 \times 9! \times 9!$

(c) $\frac{11!}{5!} \times 9! \times 9!$

(d) ${}^{11}C_5$

Solution: (b)

Given that 4 particular guests will sit on a particular side A (say)
and three other on the other side B (say). Therefore, we are to arrange 11 guests so that 5 guests will sit on side A and remaining 6 will sit on side B.

This can be done in the following ways--

${}^{11}C_5 \times {}^{6}C_6 = {}^{11}C_5$

Now 9 guests on each side can arrange among themselves in 9! ways.

Therefore, total number of arrangements

${}^{11}C_5 \times 9! \times 9!$

Question:

The sum of all four digit numbers that can be med using the digits 1, 2, 3, 4, when repetition of digits is not allowed, is

(a) 36600

(b) 66000

(c) 36000

(d) 66660

Solution: (d)

The sum of all n-digits numbers formed by using n-digits from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, is given by $(\text{Sum of digits})(n-1)!\left(\frac{10^n-1}{9}\right)$

Therefore, required sum $= (10)(3!)\left(\frac{10^4-1}{9}\right)$

$= 66660$

Question:

The locus of the centre of a circle, which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is given by the equation

(a) $x^2 - 6x - 10y + 14 = 0$

(b) $x^2 - 10x - 6y + 14 = 0$

(c) $y^2 - 6x - 10y + 14 = 0$

(d) $y^2 - 10x - 6y + 14 = 0$

Solution: (d)

We have $x^2 + y^2 - 6x - 6y + 14 = 0$

$$x^2 + y^2 - 6x - 4y - 11 = 0$$
$$x^2 - 6x + 9 + y^2 - 4y + 4 - 11 - 13 = 0$$
$$(x-3)^2 + (y-2)^2 = 24$$

Let C be the centre of the circle. So, the coordinates of the centre of the circle is C (3, 2).

Since CA and CB are perpendicular to PA and PB, CP is the diameter of the circumcircle of triangle PAB.

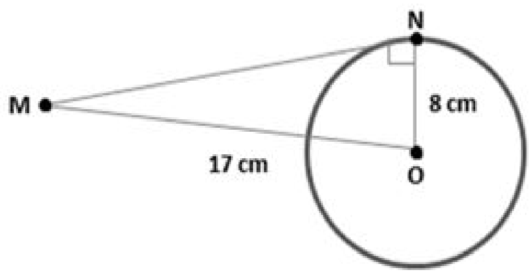
So, the equation of the circle with the end points of the diameter as (1, 8) and (3, 2) is

$$(x-1)(x-3) + (y-8)(y-2) = 0$$
$$\Rightarrow x^2 + y^2 - 4x - 10y + 19 = 0$$

Q. Find the length of the tangent from a point M which is at a distance of 17 cm from the centre O of the circle of radius 8 cm.

Sol.

Consider the figure:



Since, MN is the tangent of the circle,

$\angle MNO = 90^{0}$

$\Rightarrow MO^2 = MN^2 + ON^2$

$\Rightarrow 17^2 = MN^2 + 8^2$

$\Rightarrow 289 = MN^2 + 64$

$\Rightarrow 289 – 64 = MN^2$

$\Rightarrow MN^2 = 225$

$\Rightarrow MN = 15$

Thus, the length of the tangent is 15 cm.

Q. If the common difference of an A.P. is 3, then find $a_{20} – a_{15}$.

Sol.

Let the first term of the AP be a.

$a_n = a(n - 1)d$

$a_{20} – a_{15} = [a + (20 – 1)d] – [a + (15 – 1)d]$

$= 19d – 14d$

$= 5d$

$= 5 \times 3$

Q. Solve the following system of linear equations by substitution method:

$2x – y = 2$

$x + 3y = 15$

Sol.

Here, $2x – y = 2$

⇒ $y = 2x - 2$

⇒ $x + 3y = 15$

Substituting the value of y from (i) in (ii), we get

$x + 6x - 6 = 15$

⇒ $7x = 21$

⇒ $x = 3$

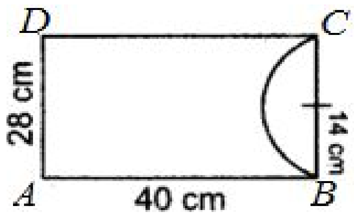
From (i), $y = 2 \times 3 - 2 = 4$

∴ $x = 3$ and $y = 4$

Q. From a rectangular sheet of paper ABCD with AB = 40 cm and AD = 28 cm, a semi-circular portion with BC as diameteris cut off. Find the area of remining paper (use $\pi = 22/7$)

Sol.

Given situation can be represented as the following diagram:



Length of paper, AB = l = 40cm

Width of paper, AD = b = 28cm

Area of paper = $l \times b = 40 \times 28 = 1120\ cm^2$

Diameter of semi-circle = 28cm

∴ Radius of semi-circle, r = 14cm

Thus, area of semi-circle = 1/2. πr^2

$= 1/2 \times 22/7 \times 14 \times 14$

$= 308 cm^2$

∴ Area of remaining paper = $1120 – 308 = 812\ cm^2$

Q. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th term is 44. Find the first three terms of the AP.

Sol.

Given that sum of the 4th and 8th terms of an AP is 24.

$\Rightarrow \quad a + 3d + a + 7d = 24$

$\Rightarrow \quad 2a + 10d = 24 \qquad \text{...(i)}$

Also the sum of the 6th and 10th term is 44.

$\Rightarrow \quad a + 5d + a + 9d = 44$

$\Rightarrow \quad 2a + 14d = 44 \qquad \text{...(ii)}$

Subtracting equation (i) from equation (ii), we get:

$\quad 4d = 20$

$\Rightarrow \quad d = 5$

Substituting $d = 5$ in equation (i), we have:

$\quad 2a + 10d = 24$

$\Rightarrow \quad 2a + 10\,(5) = 24$

⇒ $2a + 50 = 24$

⇒ $2a = -26$

⇒ $a = -13$

Hence first term of given A.P. is −13 and common difference is 5.

Q. The taxi charges in a city comprise of a fixed charge together with the charges for the distance covered. For a journey of 10 km the charge paid is Rs. 75 and for a journey of 15 km the charge paid is Rs. 110. What will a person have to pay for travelling a distance of 25 km?

Sol.

Let the fixed charge of taxi be Rs. *x* per km and the running charge be Rs *y* per km.

According to the question,

$x + 10y = 75$

$x + 15y = 110$

Subtracting equation (ii) from equation (i), we get

$-5y = -35$

$\Rightarrow y = 7$

Putting $y = 7$ in equation (i), we get $x = 5$

Therefore, Total charges for travelling a distance of 25 km = $x + 25y$

$= (5 + 25 \times 7)$

$= (5 + 175)$

= Rs 180

Q. A motor boat whose speed is 24 km/hr in still water takes 1 hr more to go 32km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol.

Let the speed of stream be x.

Then,

Speed of boat in upstream is $24 - x$

In downstream, speed of boat is 24 + *x*

According to question,

Time taken in the upstream journey − Time taken in the downstream journey = 1 hour

$$\Rightarrow \frac{32}{24-x} - \frac{32}{24+x} = 1$$
$$\Rightarrow \frac{24+x-24+x}{(24)^2 - x^2} = \frac{1}{32}$$
$$\Rightarrow \frac{2x}{576-x^2} = \frac{1}{32}$$
$$\Rightarrow x^2 + 64x - 576 = 0$$
$$\Rightarrow x^2 + (72-8)x - 576 = 0$$
$$\Rightarrow x^2 + 72x - 8x - 576 = 0$$
$$\Rightarrow x(x+72) - 8(x+72) = 0$$
$$\Rightarrow (x-8)(x+72) = 0$$
$$\Rightarrow x = 8, -72$$

Since speed cannot be negative, therefore speed of stream is 8km/hr.

Q. The first and the last terms of an AP are 10 and 361 respectively. If its common difference is 9 then find the number of terms and their total sum?

Sol.

Given, first term, a = 10

Last term, a_l = 361

And, common difference, d = 9

Now $a_l = a + (n - 1)d$

$\Rightarrow \quad 361 = 10 + (n - 1)9$

$\Rightarrow \quad 361 = 10 + 9n - 9$

$\Rightarrow \quad 361 = 9n + 1$

$\Rightarrow \quad 9n = 360$

$\Rightarrow \quad n = 40$

Therefore, total number of terms in AP = 40

Now, sum of total number of terms of an AP is given as:

$$S_n = n/2\,[2a + (n - 1)d]$$

$\Rightarrow \quad S_{40} = 40/2\,[2 \times 10 + (40 - 1)9]$

$= 20[20 + 39 \times 9]$

$=20[20 + 351]$

$=20 \times 371 = 7420$

Thus, sum of all 40 terms of AP = 7420