MATHEMATICS (PAPER-I)

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections and printed both in HINDI and in ENGLISH.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Answers must be written in the medium authorized in the Admission Certificate which must be stated clearly on the cover of this Question-cum-Answer (QCA) Booklet in the space provided. No marks will be given for answers written in a medium other than the authorized one.

Assume suitable data, if considered necessary, and indicate the same clearly.

Unless and otherwise indicated, symbols and notations carry their usual standard meanings.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.
1. (a) \text{Man lêjìbei kî A \, \text{ék} \, 3 \times 2 \, \text{aáyùh} \, \text{è} \, \text{òi} \, B \, \text{ék} \, 2 \times 3 \, \text{aáyùh} \, \text{è}. \, \text{Dàshài kale kî C = A \cdot B \, \text{èk} \, \text{áñyùkzúmányi òãàyùh} \, \text{è}.}

Let A be a 3 \times 2 matrix and B a 2 \times 3 matrix. Show that C = A \cdot B is a singular matrix.

(b) \text{Aáadhár sàdhis e}_1 = (1, 0) \text{ kò e}_2 = (0, 1) \, \text{kô} \, \alpha_1 = (2, -1) \, \text{è} \, \alpha_2 = (1, 3) \, \text{kè \, ràñik sànyòg \, kè \, rùp \, ì \, áyùk \, kë;} \, \text{këjìjîye.}

Express basis vectors e_1 = (1, 0) \text{ kò e}_2 = (0, 1) \, \text{kè \, línêr cèmibhèns \, kè \, \alpha_1 = (2, -1) \, \text{è} \, \alpha_2 = (1, 3).}

(c) \text{Nìpìàrì tì këjìjîye kî lim}_ {z \rightarrow 1} (1 - z) \tan \frac{\pi z}{2}

Determine if \lim_ {z \rightarrow 1} (1 - z) \tan \frac{\pi z}{2} \text{ exists or not. If the limit exists, then find its value.}

(d) \text{Sìmà \, lim}_ {n \rightarrow \infty} \frac{1}{n^2} \sum_{r=0}^{n-1} \sqrt{n^2 - r^2} \text{ kà \, mân \, zñàt \, këjìjîye.}

Find the limit \lim_ {n \rightarrow \infty} \frac{1}{n^2} \sum_{r=0}^{n-1} \sqrt{n^2 - r^2}.

(e) \text{Sàlì \, ðêkà \, \frac{x-1}{2} = \frac{y-1}{3} = \frac{z+1}{-1} \, \text{kà \, sàmùlùl} \, x + y + 2z = 6 \, \text{pè \, pëkùzëpùn \, ñàt \, këjìjîye.}

Find the projection of the straight line \frac{x-1}{2} = \frac{y-1}{3} = \frac{z+1}{-1} \text{ on the plane } x + y + 2z = 6.

2. (a) \text{Aágà A \, è} \, B \, \text{sàmùp} \, n \times n \, \text{aáyùh} \, \text{è}, \, \text{tò \, dàshài lèjìbei kî ùnkê kà \, òãàyùh \, mân \, èk \, hè è.}

Show that if A and B are similar \, n \times n \, \text{matrices, then they have the same eigenvalues.}

(b) \text{Bëndù} \, (1, 0) \, \text{sè \, pàkùlùp} \, y^2 = 4x \, \text{tòk \, kî \, nùùsùm \, ðùù \, ñàt \, këjìjîye.}

Find the shortest distance from the point \(1, 0)\) to the parabola \(y^2 = 4x\).
(c) The ellipse \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \] revolves about the x-axis. Find the volume of the solid of revolution.

(d) The lines
\[ a_1 x + b_1 y + c_1 z + d_1 = 0 \]
\[ a_2 x + b_2 y + c_2 z + d_2 = 0 \]
and \( z \)-axis.

Find the shortest distance between the lines
\[ a_1 x + b_1 y + c_1 z + d_1 = 0 \]
\[ a_2 x + b_2 y + c_2 z + d_2 = 0 \]
and the \( z \)-axis.

3. (a) The system of linear equations
\[ x + 3y - 2z = -1 \]
\[ 5y + 3z = -8 \]
\[ x - 2y - 5z = 7 \]

(i) The system has no solution.

(ii) The system has a unique solution.

(iii) The system has infinitely many solutions.
(b) 面值定比 \( f(x, y) = xy^2 \), \( y > 0 \)
\[ f(x, y) = -xy^2, \quad \text{if} \quad y \leq 0 \]

研究函数在 \( (0, 1) \) 处及 \( (0, 1) \) 处是否存在，以及其存在性。

Let
\[ f(x, y) = xy^2, \quad \text{if} \quad y > 0 \]
\[ = -xy^2, \quad \text{if} \quad y \leq 0 \]

Determine which of \( \frac{df}{dx} \) (0, 1) and \( \frac{df}{dy} \) (0, 1) exists and which does not exist.

(c) 轨迹 \((x + y + z)(2x + y - z) = 6z\) 的某条直线的方程求出，若点 \((1, 1, 1)\) 并且存在。

Find the equations to the generating lines of the paraboloid
\((x + y + z)(2x + y - z) = 6z\) which pass through the point \((1, 1, 1)\).

(d) \(xyz\)-轴上，给定点 \((0, 0, 0), (0, 1, -1), (-1, 2, 0)\) 和 \((1, 2, 3)\) 通过这些点的圆周求出。

Find the equation of the sphere in \(xyz\)-plane passing through the points
\((0, 0, 0), (0, 1, -1), (-1, 2, 0)\) and \((1, 2, 3)\).

4. (a) 立方体 \([2, 3] \) 上的 \(x^4 - 5x^2 + 4\) 的最大值和最小值求出。

Find the maximum and the minimum values of \(x^4 - 5x^2 + 4\) on the interval \([2, 3]\).

(b) 求定积分
\[ \int_0^a \int_{x^2 + y^2} dx \, dy \]

Evaluate the integral
\[ \int_0^a \int_{x^2 + y^2} x \, dx \, dy \]

(c) 面值定比 \((0, 0, 1)\) 且该面值的
\[ 2x^2 - y^2 = 4, \quad z = 0 \]

Find the equation of the cone with \((0, 0, 1)\) as the vertex and \(2x^2 - y^2 = 4, \quad z = 0\)
as the guiding curve.

(d) \(3x - y + 3z = 8\) 的直线及给定 \((1, 1, 1)\) 通过这些点的圆周求出。

Find the equation of the plane parallel to \(3x - y + 3z = 8\) and passing through the point \((1, 1, 1)\).
5. (a) Solve:
\[ y'' - y = x^2 e^{2x} \]
(b) \( x = 3t, \ y = 3t^2, \ z = 3t^3 \)
Samikaranon vaale vaale vab ke ek aam bindu par sparsha-reakha aur reyak
\( y = z - x = 0 \) ke beech ka kroon zaat kijiyen.
Find the angle between the tangent at a general point of the curve whose
equations are \( x = 3t, \ y = 3t^2, \ z = 3t^3 \) and the line \( y = z - x = 0 \).
(c) Solve:
\[ y'' - 6y'' + 12y' - 8y = 12 e^{2x} + 27 e^{-x} \]
(d) \( f(t) = \frac{1}{\sqrt{t}} \) ka laplaceRPantar zaat kijiyen.
\[ \frac{5s^2 + 3s - 16}{(s - 1)(s - 2)(s + 3)} \] ka vilom laplaceRPantar zaat kijiyen.
Find the inverse Laplace transform of \( \frac{5s^2 + 3s - 16}{(s - 1)(s - 2)(s + 3)} \).
(e) Ek khan ko dhartti ke ek bindu se prakshetrit kare par waha ek deewar, jo prakshetra bindu se \( d \) duri par hai aur
jiske nechh \( h \) hai, ko chhure pade par karte hai. Aman yeh khan utdharapar tala par gatiyon hai aur aise khetra
par \( R \) hai, to prakshetra ki utbhat zaat kijiyen.
A particle projected from a given point on the ground just clears a wall of
height \( h \) at a distance \( d \) from the point of projection. If the particle moves
in a vertical plane and if the horizontal range is \( R \), find the elevation of
the projection.

6. (a) Solve:
\[ \left( \frac{dy}{dx} \right)^2 + 2 \frac{dy}{dx} x - y = 0 \]
(b) Ek khan, jo ek saat reyak mein saat abart gaati se chal raha hai, ke path ke tendr sxe \( x_1 \) aur \( x_2 \) ki duri par bage
kroona: \( u_1 \) aur \( u_2 \) hain. Uske gaati ka aarberikal zaat kijiyen.
A particle moving with simple harmonic motion in a straight line has velocities
\( u_1 \) and \( u_2 \) at distances \( x_1 \) and \( x_2 \) respectively from the centre of its path.
Find the period of its motion.
(c) Solve:
\[ y'' + 16y = 32 \sec 2x \]
(d) If $S$ is the surface of the sphere $x^2 + y^2 + z^2 = a^2$, then evaluate
\[
\iint_S [(x + z) \, dydz + (y + z) \, dzdx + (x + y) \, dxdy]
\]
using Gauss' divergence theorem.

7. (a) Solve:
\[(1 + x)^2 y'' + (1 + x) y' + y = 4\cos(\log(1 + x))\]

(b) Find the curvature and torsion of the curve
\[
\vec{r} = a(u - \sin u) \vec{i} + a(1 - \cos u) \vec{j} + bu \vec{k}
\]

(c) Solve the initial value problem
\[
y'' - 5y' + 4y = e^{2t}
\]
\[y(0) = \frac{19}{12}, \quad y'(0) = \frac{8}{3}\]

(d) Find $\alpha$ and $\beta$ such that $x^\alpha y^\beta$ is an integrating factor of
\[(4y^2 + 3xy) \, dx - (3xy + 2x^2) \, dy = 0\]
and solve the equation.
8. (a) Let \( \mathbf{v} = v_1 \mathbf{i} + v_2 \mathbf{j} + v_3 \mathbf{k} \). Show that \( \text{curl}(\text{curl}\ \mathbf{v}) = \text{grad}(\text{div}\ \mathbf{v}) - \mathbf{v} \).

(b) Evaluate the line integral \( \int_C -y^3 \, dx + x^3 \, dy + z^3 \, dz \) using Stokes’ theorem. Here \( C \) is the intersection of the cylinder \( x^2 + y^2 = 1 \) and the plane \( x + y + z = 1 \). The orientation on \( C \) corresponds to counterclockwise motion in the \( xy \)-plane.

(c) Let \( \mathbf{F} = xy^2 \mathbf{i} + (y+x) \mathbf{j} \). Integrate \( (\nabla \times \mathbf{F}) \cdot \mathbf{k} \) over the region in the first quadrant bounded by the curves \( y = x^2 \) and \( y = x \) using Green’s theorem.

(d) Find \( f(y) \) such that \( (2xe^{y} + 3y^2) \, dy + (3x^2 + f(y)) \, dx = 0 \) is exact and hence solve.